4-1. Introduction

- Data that can be classified into one of several categories or classifications which is known as attribute data.
- Attribute data is any quality characteristic that can not be measured BUT counted.
- Classifications such as conforming and nonconforming are commonly used in quality control.
- Another example of attributes data is the count of defects, errors, missing parts etc....
4-1. Types of Control Charts

Attributes Control charts

4-2. Control Charts for Fraction Nonconforming

- Fraction nonconforming is the ratio of the number of nonconforming items in a population to the total number of items in that population.
- Control charts for fraction nonconforming are based on the binomial distribution.
### 4-2. Control Charts for Fraction Nonconforming

Recall: A quality characteristic follows a binomial distribution if:

1. All trials are independent.
2. Each outcome is either a “success” or “failure”.
3. The probability of success on any trial is given as $p$.
   - The probability of a failure is $(1-p)$.
4. The probability of a success is constant.

#### Binomial Distribution

- The binomial distribution with parameters $n \neq 0$ and $0 < p < 1$, is given by
  
  $$p(x) = \binom{n}{x} p^x (1-p)^{n-x}$$

- The mean and variance of the binomial distribution are
  
  $$\mu = np \quad \sigma^2 = np (1 - p)$$
4-2. Control Charts for Fraction Nonconforming

Development of the Fraction Nonconforming Control Chart

Assume
- \( n \) = number of units of product selected at random.
- \( D \) = number of nonconforming units from the sample
- \( p \) = probability of selecting a nonconforming unit from the sample.
- Then:
  \[
  P(D = x) = \binom{n}{x} p^x (1-p)^{n-x}
  \]

4-2. Control Charts for Fraction Nonconforming

Development of the Fraction Nonconforming Control Chart

- The sample fraction nonconforming is given as
  \[
  \hat{p} = \frac{D}{n}
  \]
  where \( \hat{p} \) is a random variable with mean and variance
  \[
  \mu = p \quad \sigma^2 = \frac{p(1-p)}{n}
  \]
4-2. Control Charts for Fraction Nonconforming

**Standard Given**
- If a standard value of $p$ is given, then the control limits for the fraction nonconforming are

$$UCL = p + 3 \sqrt{\frac{p(1-p)}{n}}$$

$$CL = p$$

$$LCL = p - 3 \sqrt{\frac{p(1-p)}{n}}$$

---

**No Standard Given**
- If no standard value of $p$ is given, then the control limits for the fraction nonconforming are

$$UCL = \bar{p} + 3 \sqrt{\frac{\bar{p}(1- \bar{p})}{n}}$$

$$CL = \bar{p}$$

$$LCL = \bar{p} - 3 \sqrt{\frac{\bar{p}(1- \bar{p})}{n}}$$

where

$$\bar{p} = \frac{\sum_{i=1}^{m} D_i}{mn} = \frac{\sum_{i=1}^{m} \hat{p}_i}{m}$$
4-2. Control Charts for Fraction Nonconforming

**Trial Control Limits**

- Control limits that are based on a preliminary set of data can often be referred to as trial control limits.
- The quality characteristic is plotted against the trial limits, if any points plot out of control, assignable causes should be investigated and points removed.
- With removal of the points, the limits are then recalculated.

**Example**

- A process that produces bearing housings is investigated. Ten samples of size 100 are selected.

<table>
<thead>
<tr>
<th>Sample #</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td># Nonconf</td>
<td>5</td>
<td>2</td>
<td>3</td>
<td>8</td>
<td>4</td>
<td>1</td>
<td>2</td>
<td>6</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

- Is this process operating in statistical control?
4-2. Control Charts for Fraction Nonconforming

Example

\( n = 100, \ m = 10 \)

<table>
<thead>
<tr>
<th>Sample #</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td># Nonconf.</td>
<td>5</td>
<td>2</td>
<td>3</td>
<td>8</td>
<td>4</td>
<td>1</td>
<td>2</td>
<td>6</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Fraction Nonconf.</td>
<td>0.05</td>
<td>0.02</td>
<td>0.03</td>
<td>0.08</td>
<td>0.04</td>
<td>0.01</td>
<td>0.02</td>
<td>0.06</td>
<td>0.03</td>
<td>0.04</td>
</tr>
</tbody>
</table>

\[
\bar{p} = \frac{\sum_{i=1}^{m} \hat{p}_i}{m} = 0.038
\]

4-2. Control Charts for Fraction Nonconforming

Example

Control Limits are:

\[
\text{UCL} = 0.038 + 3\sqrt{\frac{0.038(1-0.038)}{100}} = 0.095
\]

\[
\text{CL} = 0.038
\]

\[
\text{LCL} = 0.038 + 3\sqrt{\frac{0.038(1-0.038)}{100}} = -0.02 \rightarrow 0
\]
4-2. Control Charts for Fraction Nonconforming

**Example**

Interpretation of Points on the Control Chart for Fraction Nonconforming

- Care must be exercised in interpreting points that plot below the lower control limit.
  - They often do not indicate a real improvement in process quality.
  - They are frequently caused by errors in the inspection process or improperly calibrated test and inspection equipment.
4-2. Control Charts for Fraction Nonconforming

The np control chart

- The actual number of nonconforming can also be charted. Let \( n \) = sample size, \( p \) = proportion of nonconforming. The control limits are:

\[
\begin{align*}
\text{UCL} &= np + 3\sqrt{np(1-p)} \\
\text{CL} &= np \\
\text{LCL} &= np - 3\sqrt{np(1-p)}
\end{align*}
\]

(if a standard, \( p \), is not given, use \( \bar{p} \) )

4-2.2 Variable Sample Size

- In some applications of the control chart for the fraction nonconforming, the sample is a 100% inspection of the process output over some period of time.

- Since different numbers of units could be produced in each period, the control chart would then have a variable sample size.
4-2.2 Variable Sample Size

Three Approaches for Control Charts with Variable Sample Size

1. Variable Width Control Limits
2. Control Limits Based on Average Sample Size
3. Standardized Control Chart

4-2.2 Variable Sample Size

Variable Width Control Limits
- Determine control limits for each individual sample that are based on the specific sample size.
- The upper and lower control limits are

\[ p \pm 3 \sqrt{\frac{p(1-p)}{n_i}} \]

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4-2.2 Variable Sample Size

Control Limits Based on an Average Sample Size

- Control charts based on the average sample size results in an approximate set of control limits.
- The average sample size is given by
  \[ \bar{n} = \frac{\sum_{i=1}^{m} n_i}{m} \]
- The upper and lower control limits are
  \[ \bar{p} \pm 3 \sqrt{\frac{\bar{p}(1 - \bar{p})}{\bar{n}}} \]

4-2.2 Variable Sample Size

The Standardized Control Chart

- The points plotted are in terms of standard deviation units. The standardized control chart has the following properties:
  - Centerline at 0
  - UCL = 3   LCL = -3
  - The points plotted are given by:
    \[ z_i \frac{\hat{p}_i - p}{\sqrt{\frac{p(1 - p)}{n_i}}} \]
4-3. Control Charts for Nonconformities (Defects)

- There are many instances where an item will contain nonconformities but the item itself is not classified as nonconforming.
- It is often important to construct control charts for the total number of nonconformities or the average number of nonconformities for a given “area of opportunity”. The inspection unit must be the same for each unit.

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4-3. Control Charts for Nonconformities (Defects)

Poisson Distribution

- The number of nonconformities in a given area can be modeled by the Poisson distribution. Let \( c \) be the parameter for a Poisson distribution, then the mean and variance of the Poisson distribution are equal to the value \( c \).
- The probability of obtaining \( x \) nonconformities on a single inspection unit, when the average number of nonconformities is some constant, \( c \), is found using:

\[
p(x) = \frac{e^{-c}c^x}{x!}
\]

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### 4-3.1 Procedures with Constant Sample Size

#### c-chart

- **Standard Given:**
  - UCL = \( c + 3\sqrt{c} \)
  - CL = \( c \)
  - LCL = \( c - 3\sqrt{c} \)

- **No Standard Given:**
  - UCL = \( \bar{c} + 3\sqrt{\bar{c}} \)
  - CL = \( \bar{c} \)
  - LCL = \( \bar{c} - 3\sqrt{\bar{c}} \)

---

### 4-3.1 Procedures with Constant Sample Size

#### Choice of Sample Size: The \( u \) Chart

- If we find \( c \) total nonconformities in a sample of \( n \) inspection units, then the average number of nonconformities per inspection unit is \( u = c/n \).
- The control limits for the average number of nonconformities is
  - UCL = \( \bar{u} + 3\sqrt{\bar{u}/n} \)
  - CL = \( \bar{u} \)
  - LCL = \( \bar{u} - 3\sqrt{\bar{u}/n} \)
4-3.2 Procedures with Variable Sample Size

Three Approaches for Control Charts with Variable Sample Size

1. Variable Width Control Limits
2. Control Limits Based on Average Sample Size
3. Standardized Control Chart

Variable Width Control Limits

• Determine control limits for each individual sample that are based on the specific sample size.
• The upper and lower control limits are

\[ \bar{u} \pm 3 \sqrt{\frac{\bar{u}}{n_i}} \]
4-3.2 Procedures with Variable Sample Size

Control Limits Based on an Average Sample Size

- Control charts based on the average sample size results in an approximate set of control limits.
- The average sample size is given by
  \[ \bar{n} = \frac{\sum_{i=1}^{m} n_i}{m} \]
- The upper and lower control limits are
  \[ \bar{u} \pm 3 \frac{\bar{u}}{\sqrt{n}} \]

The Standardized Control Chart

- The points plotted are in terms of standard deviation units. The standardized control chart has the follow properties:
  - Centerline at 0
  - UCL = 3  LCL = -3
  - The points plotted are given by:
    \[ z_i \frac{u_i - \bar{u}}{\frac{\bar{u}}{\sqrt{n_i}}} \]
4-3.5 Dealing with Low-Defect Levels

- When defect levels or count rates in a process become very low, say under 1000 occurrences per million, then there are long periods of time between the occurrence of a nonconforming unit.
- Zero defects occur
- Control charts ($u$ and $c$) with statistic consistently plotting at zero are uninformative.

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4-3.5 Dealing with Low-Defect Levels

**Alternative**

- Chart the time between successive occurrences of the counts – or time between events control charts.
- If defects or counts occur according to a Poisson distribution, then the time between counts occur according to an exponential distribution.

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4-3.5 Dealing with Low-Defect Levels

Consideration

• Exponential distribution is skewed.
• Corresponding control chart very asymmetric.
• One possible solution is to transform the exponential random variable to a Weibull random variable using $x = y^{1/3.6}$ (where $y$ is an exponential random variable) – this Weibull distribution is well-approximated by a normal.
• Construct a control chart on $x$ assuming that $x$ follows a normal distribution.
• See Example 6-6, page 326.

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4-4. Choice Between Attributes and Variables Control Charts

• Each has its own advantages and disadvantages
• Attributes data is easy to collect and several characteristics may be collected per unit.
• Variables data can be more informative since specific information about the process mean and variance is obtained directly.
• Variables control charts provide an indication of impending trouble (corrective action may be taken before any defectives are produced).
• Attributes control charts will not react unless the process has already changed (more nonconforming items may be produced).

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4-5. Guidelines for Implementing Control Charts

1. Determine *which* process characteristics to control.
2. Determine *where* the charts should be implemented in the process.
3. Choose the proper *type* of control chart.
4. Take action to *improve* processes as the result of SPC/control chart analysis.
5. Select data-collection systems and computer software.

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Conclusion

"Quality control truly begins and ends with education",

*K. Ishikawa (1990).*